PROOF. (*i*) If $x \in J(x), x \neq z$, then by P2, $x^n \in J(z^n) = J(z), n = 1, 2, 3 \cdots$. According to P6 and Theorem 1 no interval of $J(z)$ contains the generator q of the subgroup Q of K. But there is an interval of $J(z)$ which contains z and x^n. Hence $x^n < q$.

(*ii*) Since $J(z) = z$, P4 implies that a, b is an anomalous pair if there is a u in K such that $a \in J(u)$ and $b \in J(u)$.

It may be observed that if $K = Q$, as in the example, then Theorem 3 (*i*) takes the following stronger form.

THEOREM 4. *Suppose* S *is a totally ordered semigroup containing zeroid elements and the kernel* K *of* S *is cyclic. Then* S *is Archimedean if and only if* $J(z) = z$.

PROOF. If S is Archimedean, then $J(z) = z$ according to Theorem 3. Suppose $J(z) = z$. It follows from Theorem 1 that K is an infinite subchain of S. Hence if $u \in K$, $u > z$ and m is a positive integer, there is a positive integer n such that $u^n > u^m$. Since if $a \in J(u)$, then $a^n \in J(u^n)$, it follows from P4 that $a^n > u^m$. If $u < z$, it can be shown similarly that $a^{-n} < u^{-m}$.

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Matrices whose sum is the identity matrix

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Coimbra

1. Let $A_i (i = 1, \cdots, m)$ be $n \times n$ complex matrices and let n_i denote the rank of A_i. In [1], p. 68 the following problem is posed:

If the matrices $A_i (i = 1, \cdots, m)$ *are symmetric and* $\sum_{i=1}^{m} A_i = E$ *(*E *denotes the* $n \times n$ *identity matrix), show that the following statements are equivalent:*

a) $A_i^2 = A_i \qquad (i = 1, \cdots, m)$

b) $\sum_{i=1}^{m} n_i = n$

c) $A_i A_j = 0 \qquad (i, j = 1, \cdots, m;\ i \neq j)$.

In [3] it is asked whether it is possible to drop the condition that the matrices $A_i (i = 1, \cdots, m)$ should be symmetric. In the present note we answer this question in the affirmative. So we no longer assume that the matrices $A_i (i = 1, \cdots, m)$ are symmetric.

We prove first that *a*) implies *b*).

If A_i is idempotent there exists a nonsingular matrix T_i such that (see [2], Vol. I, p. 226)

$$(1) \qquad A_i = T_i \operatorname{diag}(\overbrace{1, \cdots, 1}^{n_i}, \overbrace{0, \cdots, 0}^{n-n_i}) T_i^{-1}.$$

This can be proved very easily if we note that A_i is idempotent if and only if each diagonal block in its JORDAN normal form